Brockwell Junior School

Progression in Calculation

A message for Parents/Carers: What you need to know about calculations

Mathematics will be at the core of your child's schooling from the moment they start to the moment they leave. They will be involved in drawing, measuring, handling data and many other practical activities that will help your child to understand and enjoy the subject. This booklet offers guidance to the methods used to help our pupils with calculations. The methods we are advocating are in line with the National Curriculum. We hope this will be helpful to you and that you will be able to support your child in learning by heart the basic rules which will assist in mental recall e.g. number bonds and multiplication tables.

The methods that we use in school may or may not be familiar to you. Children are often confused when they ask parents for help at home and they try to teach the methods that *they* themselves were taught. Knowing how the methods in this booklet work, will help you to help your children. All staff in school work from this document so that we can ensure the consistency of our approach and can make sure that the children move onto the next step when they are ready.

The four operations that are covered by this booklet are addition, subtraction, multiplication and division. At times, addition and subtraction will be taught alongside each other so that children can see the link between them (inverse). The same link will apply to the teaching of multiplication and division. As well as this, there is a supplementary section on working with fractions. Whichever operation is being taught, the child needs to experience all of these steps to completely conquer it. The idea is that children use concrete apparatus, moving onto pictorial representations and finally the abstract (just the numbers themselves). As much as possible, we will teach maths in context and link to real life situations. We will also use the most appropriate written and mental calculations when carrying out calculations involving money, weight, length and capacity.

1) using objects
2) using pictures
3) using a numberline
4) using an expanded method
5) using a compact written method

Mental Methods First

Children should always be encouraged to consider if a mental calculation would be appropriate before using written methods. These are covered in the first part of each section. When working out mental calculations, children will usually be taught to work with the numbers with the largest value first. E.g. 72+36 could be worked out by:

- 72 + 30 = 102 + 6 = 108 or
- 70 + 30 = 100, 6 + 2 = 8 so 72 + 36 = 108

Children will be taught both methods and they will choose the best for them. They may also use their own methods for solving, as long as they are efficient. Methods will be modelled to others and encouraged to be used.

Why do children need to do written calculations?

- To represent work that has been done practically.
- To support, record and explain mental calculation.
- To keep track of steps in a longer task.
- To work out calculations that are too difficult to do mentally.

Children should be taught when it is appropriate to do an approximate or estimate for the calculation using a known mental method.
Children will be taught that addition is the inverse of subtraction and that multiplication is the inverse of division. They will use the inverses to check their calculations are accurate.

By upper Key Stage 2, children should be confident in choosing and using a strategy that they know will get them to the correct answer as efficiently as possible.

Please note: If, after much practise, some children find the written method too difficult to understand, alternative methods will be taught to these children so that all children have the same opportunities to be fluent in number.

What can you as parents do to help?

- Count with your child.
- Practise number bonds within 10 and within 20
- Learn times tables so that pupils have rapid recall. The expectation is that pupils know all of their tables up to 12 x 12 by the end of Y4.
- Play number games.
- Involve children when taking measurements or weighing items.
- Take note of numbers in real life e.g. telephone numbers, bus numbers, lottery numbers etc.
- Give children opportunities to use money to shop, check change etc.
- Talking about the mathematics every-day situations. For example, football e.g. 'How many points does your favourite team need to catch the next team in the league?'
- When helping your child calculate, use the method that the child has been taught.

Please don't…

- Teach your children that to multiply by 10 you 'just add a zero'. – you 'move the digits to the left and add a zero as a place holder'
- Teach the methods that we don't use at school. Please feel free to speak to your child's class teacher or the Maths Subject Leader should you wish to discuss this.
- Tell them that you can move the decimal point. – You can't. You can only move the digits to the left or to the right and decimal point stays in the same place
- Tell them that they are doing 'sums' – 'sum' is a mathematical word that means 'addition', everything else is a 'calculation'

Glossary

2-digit: A number with 2 digits like 23, 45, 12 or 60**.**

3-digit: A number with 3 digits like 123, 542 or 903.

Addition facts: Knowing that 1 + 1 = 2 and 1 + 3 = 4 and 2 + 5 = 7. Normally we only talk about number facts with totals of 20 and under.

Array: An array is an arrangement of a set of numbers or objects in rows and columns – it is mostly used to show how you can group objects for repeated addition, multiplication and division.

Bridge to ten: A strategy when using number lines. Adding a number that takes you to the next 'tens' number.

Bus Stop Method: Traditional method for division with a single or two digit divisor.

Concrete apparatus: Objects to help children count – these are most often cubes (multilink) but can be anything they can hold and move. Dienes (hundreds, tens and units blocks), Numicon, Cuisenaire rods are also referred to as **concrete apparatus**.

Decimal number: A number with a decimal point.

Divisor: The smaller number in a division calculation**.**

Double: Multiply a number by 2.

Exchanging and regrouping: Moving a 'ten' or a 'hundred' from its column into the next column and splitting it up into ten 'ones' or ten 'tens' and putting it into a different column.

Expanded Multiplication: A method for multiplication where each stage is written down and then added up at the end in a column.

Find the difference: A method for subtraction involving counting up from the smaller to the larger number.

Grid method: A method for multiplying two numbers together involving partitioning.

Grouping: Putting objects into groups of a certain size.

Half: A number, shape or quantity divided into 2 equal parts**.**

Halve: Divide a number by 2.

Integer: A number with no decimal point (whole number).

Inverse: The opposite operation. Addition is the inverse of subtraction, multiplication is the inverse of division.

Long Multiplication: Column multiplication where only the significant figures are noted.

Number bonds to ten: 2 numbers that add together to make ten, like 2 and 8, or 6 and 4.

Number bonds to 100: 2 numbers that add together to make 100 like 20 and 80, or 45 and 65 or 12 and 88.

Number line: A line either with numbers or without (a blank number line). Children use this tool to help them count on for addition or subtraction and also in multiplication and division.

Number line Chunking: Method of division involving taking chunks or groups or the divisor away from the larger number.

Number sentence: Writing out a calculation with just the numbers in a line E.g. 2 + 4 = 6 or 35 ÷ 7 = 5 or 12 x 3 = 36 or 32 – 5 = 27

Partition: Split up a larger number into the hundreds, tens and ones. E.g. 342 is 300 and 40 and 2.

Place Value: Knowing that in the number 342 – the '3' means '3 hundreds', the '4' means '4 tens' and the '2' means '2 ones'.

Quarter: A number, shape or quantity divided into 4 equal parts

Recombine: For addition, once you have partitioned numbers into hundreds, tens and units then you have to add the hundreds together, then add the tens to that total, then add the units to that total.

Remainder: A whole number left over after a division calculation

Repeated addition: Repeatedly adding groups of the same size for multiplication (e.g. 6 + 6 + 6 = 18 this would link to 6 X 3 = 18).

Significant digit: The digit in a number with the largest value. E.g. in 34: the most significant digit is the 3, as it has a value of '30' and the '4' only has a value of '4'.

Single digit: A number with only one digit. These are always less than 10.

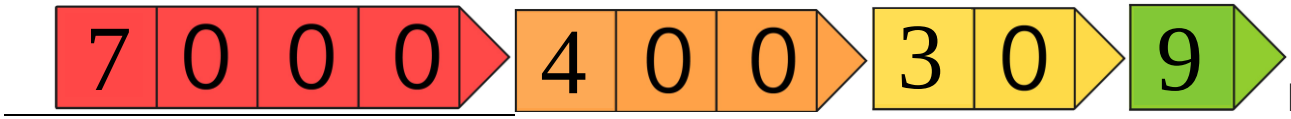
Taking away: A method for subtraction involving counting backwards from the larger to the smaller number.

Tens number: A number in the ten times tables – 10, 20, 30 etc.

Unit: Another term for single digit numbers. The right hand column in column methods is the 'units' column or the 'ones' column.

Resources that your children will use to help with calculation

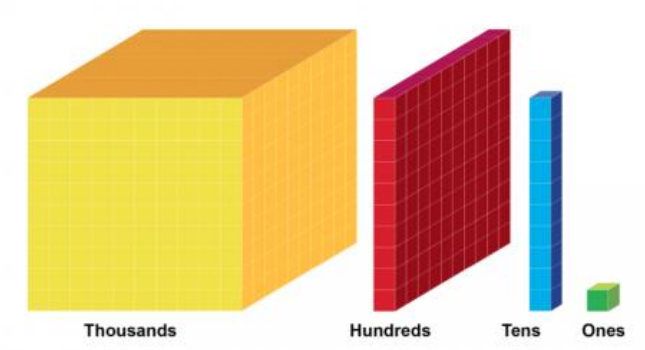
Place Value/Arrow Cards



These help children when **partitioning** numbers and working out the **place value** of numbers.

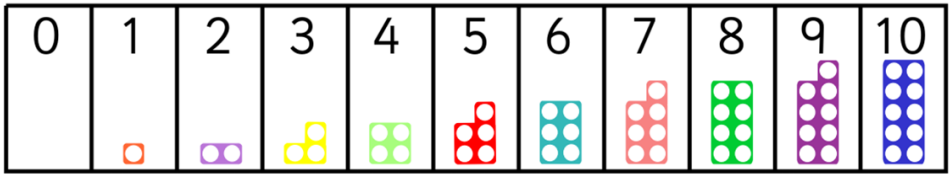
Place Value Counters

Dienes



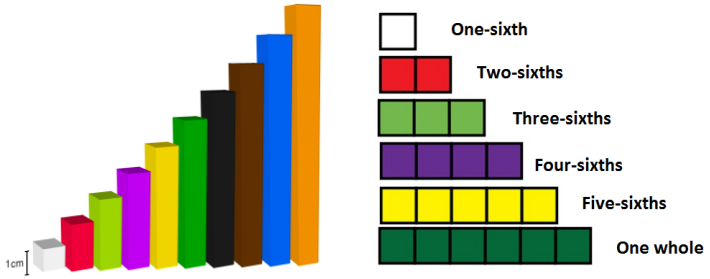
Although it has been used in schools for years, it is still a crucial step in knowing what a one, a ten, a hundred and a thousand look like and how they can be added together and split up to form smaller and larger numbers. This concrete apparatus will be used alongside written calculations to support children's understanding.

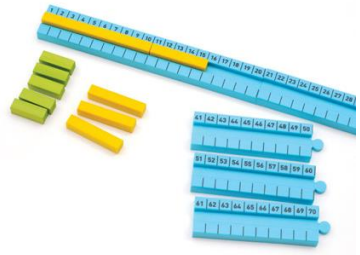
Numicon



Numicon is an especially useful resource as it can be used for teaching all four operations as well as fractions, decimals, percentages and a range of other aspects of maths. Each piece represents an integer from 1 to 10. The children love using it as it is colourful and tactile.

Cuisenaire Rods and Number Rods

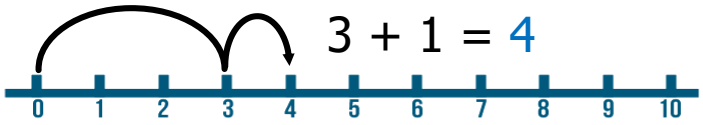




These little rods usually represent integers from 1 to 10 but they can also be used to represent other numbers. They are really useful for all the number operations as well as looking at fraction and decimal equivalents.

Number lines

Number lines are used for teaching calculations. We have pre-numbered and blank number lines in school that children can write on, or they can draw their own as appropriate for the calculation.



Hundred Square

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiplication Square

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

Age Appropriate Counters

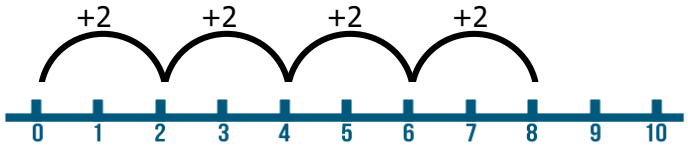
It is important that all children have access to age appropriate counters to help them with their maths calculations.

Progression through Addition

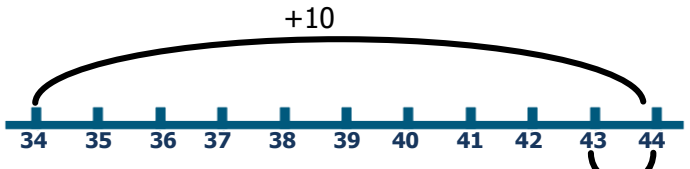
Vocabulary + Add Addition Plus And Count on More than Sum Total Altogether Increase

Knowledge

- Addition can be done in any order, *e.g. 34 + 56 = 90 or 56 + 34 = 90*.
- Usually start with the largest number (so that you have to do less counting and so there is less potential for mistakes.) *e.g. 27 + 5 = 32*.
- Must know **number bonds to 10**, *e.g.* 1 + 9 = 10, 2 + 8 = 10, 3 + 7 = 10, 4 + 6 = 10, 5 + 5 = 10 etc.
- **Addition facts** for all **single-digit** numbers, *e.g. 1 + 1 = 2, 1 + 2 = 3, 1 + 4 = 5, 2 + 1 = 3, 2 + 2 = 4, 2 + 3 = 5 etc.*
- Count forward in steps of 1, 2, 5, 10 and 100 along a **number line**.



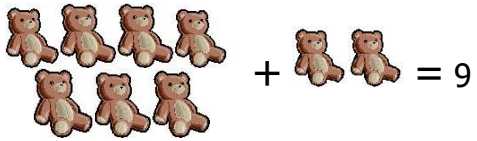
- Understand the **number line** as a continuum. A **number line** is just a tool that helps us count forwards and backwards – it has no 'official' starting or ending point.
- **Concrete apparatus** available, *e.g. using objects like multilink, Dienes, toys, blocks, Cuisenaire rods, Numicon***.**
- Understand **place value**, *e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7 ones/units.*
- Can **partition** number, *e.g. Can split a number like 327 into 300 + 20 + 7*
- Counting forwards and backwards in steps of different sizes. *e.g. counting forwards in ones 1,2,3,4,5 etc; or in steps of two 2,4,6,8,10 etc; or in steps of five 5,10,15,20,25 etc; or in steps of ten 10,20,30,40,50 etc.*
- Know doubles of numbers from 1-10 *e.g. double 3 is 6, (or 2 lots of 3 is 6, or 2 times 3 is 6, or 2 groups of 3 is 6*).
- Know doubles of numbers from 10-20, *e.g. double 12 is 24, (or 2 lots of 12 is 24, or 2 times 12 is 24, or 2 groups of 12 is 24).*
- Know that adding numbers (above zero) always produces a larger answer.
- Know that addition can be calculated in any order, *e.g. 2+3=5 or 3+2=5*
- If adding 9, add 10 and adjust by taking away 1 and show on a number line, e.g.



Progression through Addition

Skills

Non-standard written methods

1. Count up to 10 objects reliably (using apparatus) and then up to 20.  = 9

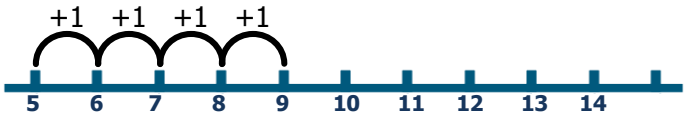
2. Find 'one more' than a number. *e.g. when given a number, they can count on to find 'one more' e.g. one more than 13 is 14*

3. Add two or more groups of objects together to find a total of less than 10. These may be **concrete apparatus** or pictures.

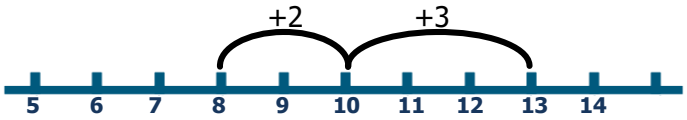
 = 7

4. Use the + and = signs to record mental calculations in a **number sentence**. *e.g. 2 + 6 = 8*

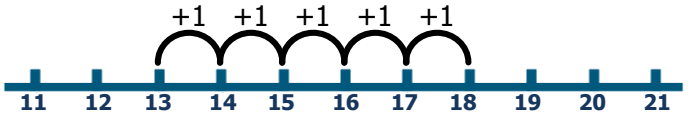
5. Count along a **numberline** to add **single digit** numbers together to find a total of less than 10 *e.g.* 5 + 4 = 9 Move onto numbers up to 10 in the same way



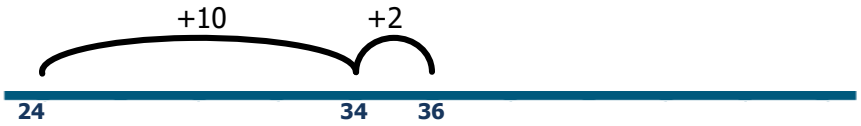
6. Add **single digit** numbers that **bridge to 10** using a **numberline**. This involves partitioning the smaller number in to 2 parts, one of which will add to the larger number to make 10 *e.g. 8 + 5 = 13*



7. Add a **2-digit** and a **single digit** number using a **numberline** *e.g. 13 + 5 = 18*

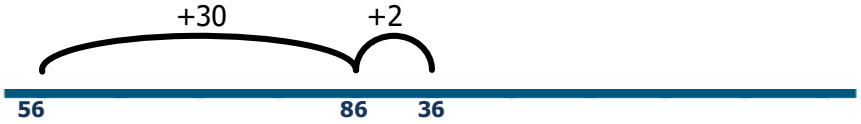


8. Add two **2-digit** numbers **bridging to 10** using a **numberline**. *e.g. 24 + 12 = 36 and show the partitioning e.g. 24 + (10+2) = 36*



9. Add two **2-digit** numbers adding the most **significant digit** first using a **blank numberline**. *e.g. 56 + 32 = 88 and show the partitioning e.g. 56 + (10 + 10 + 10 + 2) = 88*

10. Add two **2-digit** numbers adding the most **significant digit** first using a **blank numberline**. *e.g. 56 + 32 = 88*



Brockwell Junior School CALCULATION POLICY ~ PROGRESSION THROUGH ADDITION Standard Written Methods

The more formal written examples below are in year groups to match in with the objectives from the National Curriculum (written in bold)

Please Note:

- **Calculations should always be written alongside the use of concrete apparatus using the place value sheets.**
- **Concrete apparatus and pictorial representations should be used as appropriate (See Y3 for example).**

Y2 and below

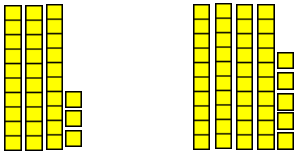
Add numbers using concrete objects, pictorial representations, including:

- ***a two-digit number and units/ones.***
- ***a two-digit number and tens.***
- ***two, two-digit numbers.***

E.g. 12 + 7 This should be shown vertically.

1 2
+ 7
1 9

E.g. 23 + 35



2 3
+ 3 5
5 8

Move onto 'carrying' and show with apparatus and pictorially with regrouping and exchanging.

E.g.

2 7
+ 3 5
6 2
1

Y3

Add numbers with up to three digits, using formal columnar written methods, with concrete objects to support regrouping/exchanging.

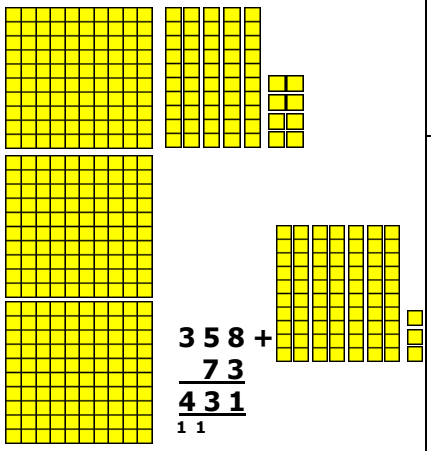
1) 3 4 2 + 1 3 3 = 4 7 5

3 0 0 + 4 0 + 2
1 0 0 + 3 0 + 3
4 0 0 + 7 0 + 5 + 4 7 5

2) 3 5 8 + 6 4 = 4 2 2

3 0 0 + 5 0 + 8
6 0 + 4
4 0 0 + 2 0 + 2 = 4 2 2
1 0 0 1 0

E.g. 358 + 73



3 5 8 +
7 3
4 3 1
1 1

Estimate answers to calculations; use inverse operations to check answers.
Children should estimate answers using appropriate mental methods.
Known methods for subtraction should be used as an inverse operation.

Y4

Add numbers with up to 4 digits using the formal written methods of columnar addition.

E.g.

2 3 5 8 +
1 7 3
2 5 3 1
1 1

Estimate and use inverse operations to check answers to a calculation.

Solve + & - two-step problems in contexts, deciding which operations and methods to use & why.

Y5

Add whole numbers with more than 4 digits, including using formal columnar methods

E.g.

4 2 3 5 8 +
4 1 7 3
4 6 5 3 1
1 1

Add numbers mentally with increasingly large numbers.

Use rounding to check answers and determine, in the context of a problem, levels of accuracy.

Solve addition multi-step problems in contexts, deciding which operations and methods to use & why.

Y6

Solve addition multi-step problems in contexts, deciding which operations and methods to use and why

Solve problems involving addition.

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Pupils should use the same methods to solve calculations involving decimals

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division.

789 + 642 becomes

7 8 9 +
6 4 2
1 4 3 1
1 1

Answer: 1431

**For interactive representations try mathsframe.co.uk ~ Resources ~ New Mathematics Curriculum (2014) / Addition & Subtraction

Progression through Subtraction

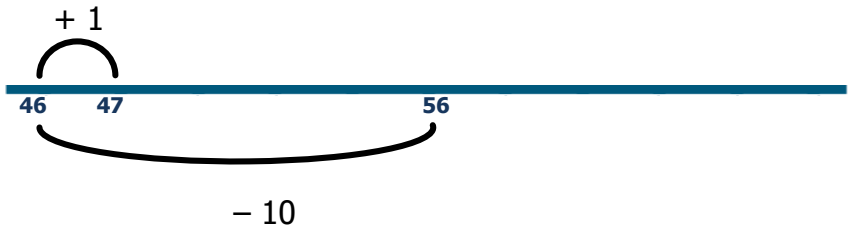
Vocabulary – Subtract Take Away Minus Less than Less Fewer Difference Decrease

Subtraction can be seen in two ways: as 'taking away/counting back' or as 'finding the difference/counting on'.

Knowledge

- '**Taking away'** is usually used when subtracting a small number from a much larger one; usually **2-digit** subtract a **single digit** like 32 – 6. This is sometimes called 'counting back.'
- Must know **number bonds** to 10 and the reverse, *e.g. 1 + 9 = 10, 2 + 8 + 10, 3 + 7 = 10 etc and 10 – 1 = 9, 10 – 2 = 8, 10 – 3 = 7 etc*.
- Must know **number bonds** to 100 (sometimes called **complements** to 100) *e.g. 20 + 80 = 100, 45 + 55 = 100, 100 – 43 = 57, etc*.
- Understand the **number line** as a continuum. A **number line** is just a tool that helps us count forwards and backwards – it has no 'official' starting or ending point.
- Unlike with addition, subtraction cannot be calculated in any order. *e.g. 9 – 4 = 5 is not the same as 4 – 9 = -5*.
- Understand **place value**, *e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means*.
- Addition and subtraction inverses e.g. 6 + 5 = 11 5 + 6 = 11 11 – 6 = 5 11 – 5 = 6.
- In mental work, when subtracting 9, subtract 10 and add 1 (This could be shown on a number line).

e.g. 56 – 9 = 47



Progression through Subtraction

Skills

Non-standard written methods

1. Use **concrete apparatus** to physically '**take away'** from numbers up to 10 and then up to 20. Imagine you have 7 teddies and you want to take away 3 of them. For this calculation you would have 7 teddies and physically take 3 away from them.

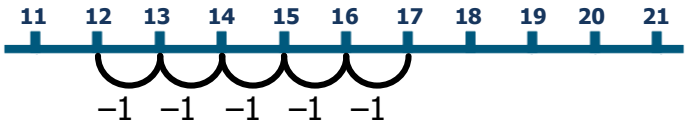
2. Find 'one less' than a number, *e.g. when given a number, they can count back to find 'one less' e.g. one less than 13 is 12.*

3. Use the – and = signs to record mental calculations in a **number sentence**. *e.g. 6 – 2 = 4*

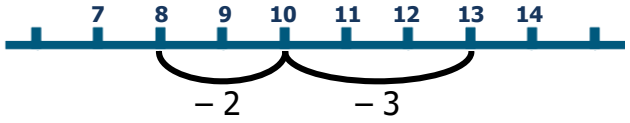
4. **When counting back, count underneath the number line** Count back on a **number line** to subtract **single digit** numbers less than 10

   e.g. 9 – 2 = 7

5. Count back on a **number line** to subtract **single digit** numbers less than 20 *e.g.* 17 – 5 = 12

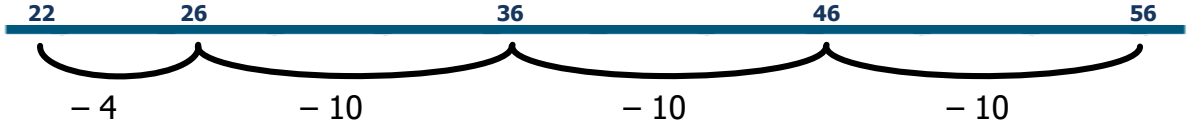


6. Subtract **single digit** numbers that **bridge to 10** using a **number line**. This involves partitioning the smaller number into 2 parts, one of which will be subtracted from the larger number to make 10 *e.g. 13 – 5= 8 (13 – 3 – 2 = 8)*

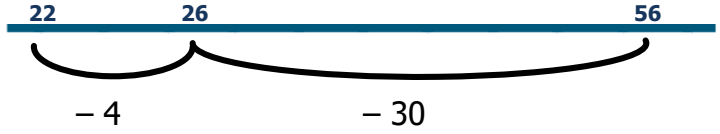


7. Use '**counting back'** – with a blank number line for larger two digit numbers e.g. 78 – 12 = 66 *and show the partitioning*
   e.g. 78 – (10 + 2) = 66

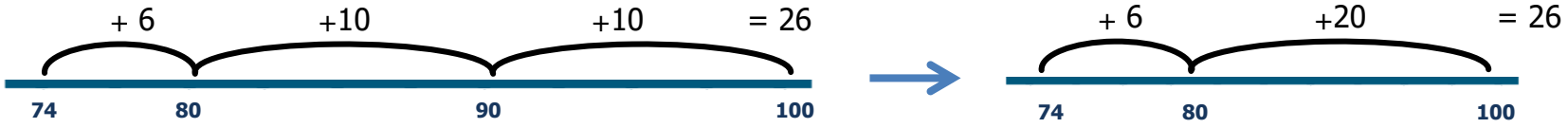
8. Use '**counting back'** – with a blank number line for larger two digit numbers e.g. 56 – 34 = 22 *and show the partitioning*
   e.g. 56 – (10 + 10 + 10 + 4) = 22



9. Use '**counting back'** – with a blank number line for larger two digit numbers e.g. 56 - 34 = 22



10. **'Finding the difference'** by counting on. By using a **number line**, count from a smaller number to a larger one.
    e.g. 100 – 74. Start at 74 and count on to 100. The 'difference' is the answer. **When counting on, count <u>above</u> the line.**



Brockwell Junior School CALCULATION POLICY ~ PROGRESSION THROUGH SUBTRACTION Standard Written Methods

The more formal written examples below are in year groups to match in with the objectives from the National Curriculum (written in bold)

Please Note:

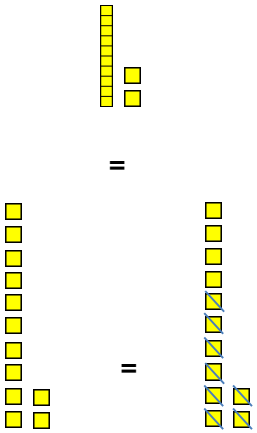
- **Calculations should always be written alongside the use of concrete apparatus using the place value sheets**
- **Concrete apparatus and pictorial representations should be used as appropriate (See Y3 for example)**

Y2 and below

Subtract numbers using concrete objects, pictorial representations, including:

- ***a two-digit number and units/ones.***
- ***a two-digit number and tens.***
- ***two, two-digit numbers.***

E.g. 12 – 8
(through regrouping/exchanging and removing ones)



Y3

Subtract numbers mentally, including:

- **a three-digit number and ones**
- **a three-digit number and tens**
- **a three-digit number and hundreds**

Subtract numbers with up to three digits, using formal columnar written methods, with concrete objects to support exchanging/regrouping.

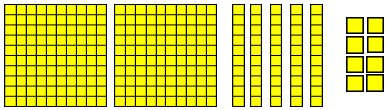
1) 254 – 121 = 133

200 50 4
100 20 1
100 30 3 = 133

2) 367 + 128 = 239

300 ^{5}60 ^{1}7
100 20 8
300 30 9 = 239

E.g. 258 – 73



By removing 2 ones, exchanging one 100 square for 10 sticks & removing 7 sticks.

$^{1}2^{1}58 -$
73
185

Estimate answers to calculations; use inverse operations to check answers

Solve problems, including missing number problems, using number facts, place value & more complex subtraction.

Y4

Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction.

E.g. 2358 – 173

$2^{2}3^{1}58 -$
173
2185

Estimate and use inverse operations to check answers to a calculation.

Solve + & - two-step problems in contexts, deciding which operations and methods to use & why.

Y5

Add whole numbers with more than 4 digits, including using formal columnar methods

E.g. 42358 – 4173

$^{1}1^{1}2^{2}3^{1}58 -$
4173
8185

Subtract numbers mentally with increasingly large numbers.

Use rounding to check answers and determine, in the context of a problem, levels of accuracy.

Solve subtraction multi-step problems in contexts, deciding which operations and methods to use & why.

Y6

Solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why

Solve problems involving subtraction.

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Pupils should use the same methods to solve calculations involving decimals

Mathematics Appendix 1: Examples of formal written methods for addition, subtraction, multiplication and division.

874 – 523 becomes

874 –
523
351

Answer: 351

932 – 457 becomes

$^{8}9^{1}2^{1}32 -$
457
475

Answer: 457

**For interactive representations try mathsframe.co.uk ~ Resources ~ New Mathematics Curriculum (2014) / Addition & Subtraction

Progression through Multiplication

Vocabulary	X	Lots of	Groups of	Times	Multiply	Multiplication	Multiple	Product	Double	Twice	Three Times
	Repeated Addition (Array column, row)										

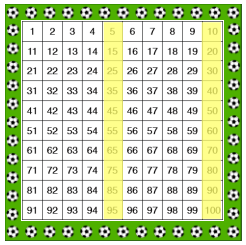
Knowledge

- Understand **place value**. *e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7*
- Recognise simple sequences of numbers. *e.g.* 5, 10, 15, 20 (*add five each time or count in 5s) 2, 4, 6, 8 (add 2 each time or count in 2s)*
- Be able to use a method for adding and subtraction (see previous sections)
- Know that multiplication can be calculated in any order *e.g. 3 x 4 = 12 and 4 x 3 = 12*
- That multiplication and division are **inverse** of each other. *e.g. 2 x 6 = 12 and 12 ÷ 6 = 2*
- Can **double** and **halve** numbers from 1 to 100 *e.g. Double 4 is 8, 4 x 2 = 8; half of 8 is 4, 8 ÷ 2 = 4*
- Multiplication is **repeated addition**. *e.g. To find 4 x 3, you add 4 groups of 3, or you add 3 four times: 3 + 3 + 3 + 3 = 12*

Progression through Multiplication

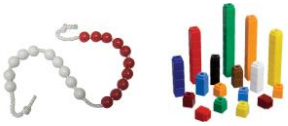
Skills

1. Counting out loud in jumps



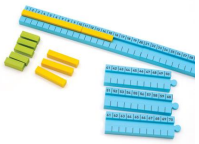
2. Counting on in multiples using a hundred square to colour in jumps of 2s, 10s, 5s etc.

3. Use resources such as bead strings and unifix to show grouping

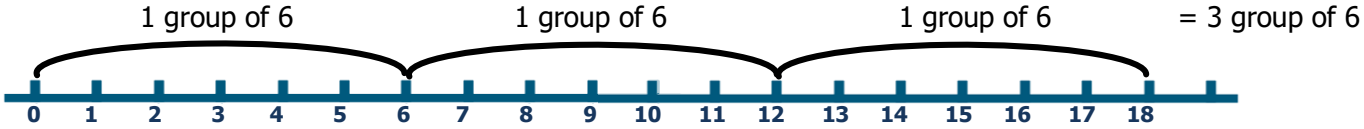


4. Pictorial *e.g. There are 3 sweets in one bag. How many sweets are there in 5 bags?*

5. Use Cuisenaire rods on a number line or number track or Numicon to multiply using **repeated addition and the link between multiplication**. *e.g. 4 + 4 + 4 + 4 + 4 = 20*



6. Use a numbered number line to count in jumps to show repeated addition and the link between multiplication

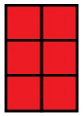


7. Be able to show multiplication facts using arrays. You can show a number, e.g. 6, in several ways using pictures or objects.

2 rows of 3 are 6

3 rows of 2 are 6

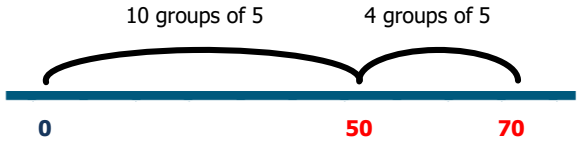
8. Arrays can be linked to the use of Cuisenaire rods where children will see that 2 rows of 3 equals the same amount as 3 rows of 2.



3 x 2 = 6

2 x 3 = 6

9. Use **times tables facts** to make more efficient jumps on a blank **number line** *e.g. for 14 x 5, you could partition and add 10 x 5 to 4 x 5*



Brockwell Junior School CALCULATION POLICY ~ PROGRESSION THROUGH MULTPLICATION

The more formal written examples below are in year groups to match in with the objectives from the National Curriculum (written in bold)

Please Note:

- **Calculations should always be written alongside the use of concrete apparatus using the place value sheets as appropriate**
- **Concrete apparatus and pictorial representations should be used as appropriate**

Y3

Use steps from previous page to teach the objectives below eg steps, 6, 7, 8

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental, grid method and progressing to formal written methods.

Solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects.

Y4

Recall multiplication facts up to 12 x 12.
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout, supported by grid method.
e.g. 23 x 6 through grid method

x	6
20	120
3	18
	138

to formal written methods (Ladder Method)

```
  23
x  6
  18  (6 x 3)
+ 120 (6 x 20)
 138
```

```
  23
x  6
 138
 1 1
```

- *Move to compact method above (only if children are ready – they must understand what the method is showing and be able to explain it. Do not teach as a method only)*
- *Numbers carried when adding should go below the answer line.*

Solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit.
e.g. a knife costs 70p and a fork cost 30p, how much for 5 sets of knives and forks? (70p + 30p) x 5 = (70 x 5) + (30 x 5)

Pupils write statements about the equality of expressions (for example, use the distributive law 39 × 7 = 30 × 7 + 9 × 7 and associative law (2 × 3) × 4 = 2 × (3 × 4)). They combine their knowledge of number facts and rules of arithmetic to solve mental and written calculations for example, 2 x 6 x 5 = 10 x 6 = 60.

Y5

Multiply numbers up to 4 digits by a one or two-digit number using a formal method, including long multiplication for two-digit numbers.

Use Grid method if needed for up to 2 digit x 2 digit and 3 digit x 1 digit. (See Y4 opposite)

Then move onto ladder method for up to 2 digit x 2 or 3 digit numbers

```
  23
X 35
  15 (5x3)
 100 (5 x 20)
  90 (30x3)
+ 600 (30x20)
 805
```

Next, move onto formal long multiplication (showing calculations in brackets)

```
  2 4 3
x   1 5
  1 2 1 5 (5 x 243)
    2 1
+ 2 4 3 0 (10 x 243)
  3 6 4 5
```

NB: Multiplying units first.
Place 0 before multiplying by 10 and explaining why this is done
Numbers carried when multiplying should be crossed off when added on so there is no confusion later in the calculation

Multiply numbers mentally drawing upon known facts

Multiply whole numbers and those involving decimals by 10, 100 & 1000.

Y6

Solve problems involving + − x ÷ and a combination of these, including understanding meaning of = sign

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication (including decimals)
e.g.

```
   1326
x    28
  10608 (8 x 1326)
  2 2 4
+ 26520 (20 x 1326)
    1
  37128
  1
```

Solve problems involving addition, subtraction, multiplication and division

Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

**For interactive representations try mathsframe.co.uk ~ Resources ~ New Mathematics Curriculum (2014) / Addition & Subtraction

Progression through Division

Vocabulary ÷ Lots of Groups of Share Halve Divide Division Divided By Divisible Remainder Factor Decimal Decimal Place

Knowledge

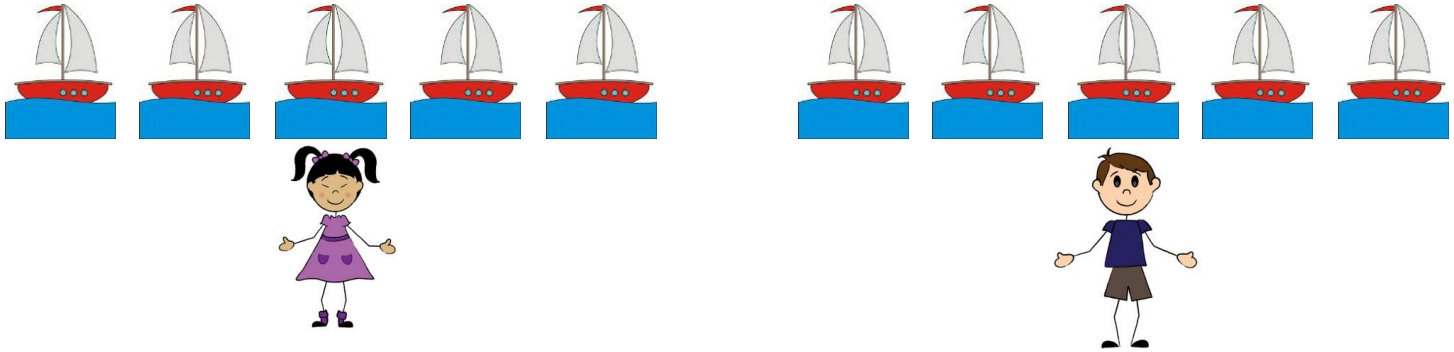
- Understand **place value**. *e.g. Knows that in the number 327, the '3' means '3 hundreds', the '2' means '20' and the '7' means 7*
- Put objects into groups of the same number.
- Recognise simple sequences of numbers. *e.g.* 5, 10, 15, 20 (*add five each time or count in 5s) 2, 4, 6, 8 (add 2 each time or count in 2s)*
- Be able to use a method for adding and subtraction (see previous sections)
- Recall multiplication facts up to 12 x 12 and derive division facts. *e.g. 5 x 4 = 20, so 20 ÷ 5 = 4 and 20 ÷ 4 = 5*
- That multiplication and division are the **inverse** of each other. *e.g. 2 x 6 = 12 and 12 ÷ 6 = 2*
- Know that division cannot be calculated in any order *e.g. 12 ÷ 4 = 3 is not the same as 12 ÷ 3 = 4*
- Know that grouping and sharing are not the same. To begin with children will group, rather than share.

Progression through Division

Skills

1. **Grouping** using apparatus e.g. 12 teddies split into 3 groups of 4

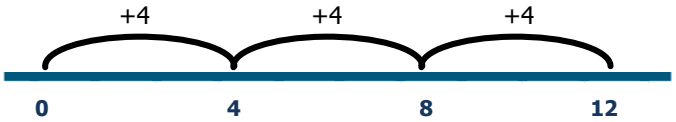
2. **Sharing** using apparatus e.g. If 10 boats are shared equally into 2 groups, how many in each group? You would physically share out the boats – one for me, one for you…



3. Use dots/pictures and circles on paper *e.g. 24 ÷ 6 = 4*

4. Division on a numberline, eg 12 ÷ 4 = 3

This shows that these are 3 lots of 4 in 12. This will be taught alongside step 3 above to begin with to show the link

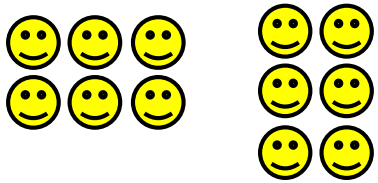


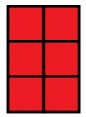
5. Use arrays to show the link between multiplication and division and write multiplication and division sentences which show the link.

E.g. 3 x 2 = 6, 2 x 3 = 6, 12 ÷ 6 = 2, 12 ÷ 2 = 6

Arrays can be linked to the use of Cuisenaire rods where children will see that 2 rows of 3 equals the same amount as 3 rows of 2.

They can then divide these up to write corresponding division facts – with arrays and Cuisenaire





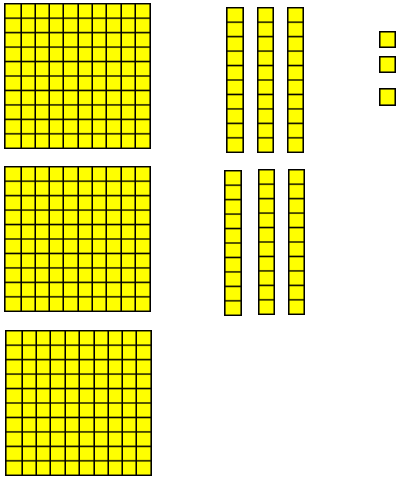
3 x 2 = 6

2 x 3 = 6

Examples of how apparatus can be used with bus stop method (at any stage) to help with division.

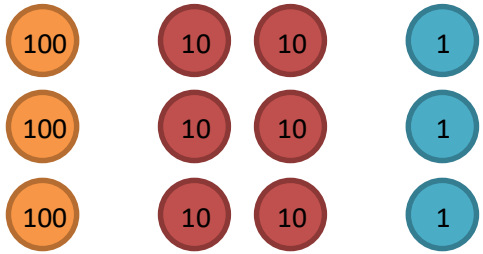
363 ÷ 3 =

$$\begin{array}{r} 121 \\ 3\overline{)363} \end{array}$$



363 ÷ 3 =

$$\begin{array}{r} 121 \\ 3\overline{)363} \end{array}$$



Brockwell Junior School CALCULATION POLICY ~ PROGRESSION THROUGH DIVISION

The more formal written examples below are in year groups to match in with the objectives from the National Curriculum (written in bold)

Please Note:

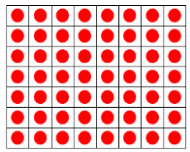
- **Calculations should always be written alongside the use of concrete apparatus using the place value sheets as appropriate**
- **Concrete apparatus and pictorial representations should be used as appropriate**

Y3

Recall & use ÷ facts for the 3, 4 and 8 tables.
Write and calculate statements for ÷ using tables they know, progressing to formal written methods.
E.g. 4 x 3 = 12 so
12 ÷ 3 = 4 & 12 ÷ 4 = 3

An image for 56 ÷ 7

8
7

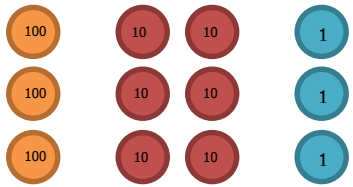


Solve problems, including missing number problems, involving division.

Y4

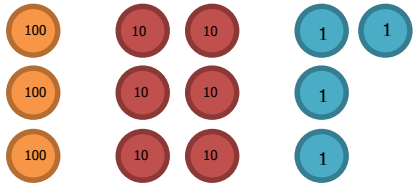
Recall division facts up to 12 x 12.
Divide two-digit and three-digit numbers by a one-digit number using formal written layout supported by concrete representation.
With exact answers

363 ÷ 3 = 121 (3 | 363)



or with simple remainders.

364 ÷ 3 = 121 r1 (3 | 364)



Dienes could be used instead of place value counters in the same way (see previous page and below right for example of how to use)

Y5

Divide numbers mentally drawing upon known facts
Divide numbers up to 4 digits by a one-digit number using the formal written method of short division; interpret remainders appropriately for the context.
E.g.

0 5 6 r2
7 | $3^{3}9^{4}4$

or

0 5 6 2/7
7 | $3^{3}9^{4}4$

or rounding the remainders to the nearest whole number e.g. how many buses? or how many full boxes?
Divide whole numbers and those involving decimals by 10, 100 & 1000.
Solve problems involving + − x ÷ and a combination of these, including understanding meaning of = sign

Y6

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of short division where appropriate, interpreting remainders according to the context.
E.g.

0 3 2
12 | $3^{3}8^{2}4$

Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division and interpret remainders as whole number remainders, fractions or by rounding, as appropriate for the context
NB: Formal Long division method was considered not needed but short division with jottings adopted instead.
E.g.

0 2 7 r2
36 | $9^{9}7^{25}7$

36
72
108
144
180
216
252
288

Solve problems involving division
Use estimation to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Mathematics Appendix 1: Examples of formal written methods for division.

432 ÷ 5 becomes

8 6 r 2
5 | 4 3 $^{3}2$

Answer: 86 remainder 2

574 ÷ 15 becomes

3 8
15 | 5 $7^{12}4$

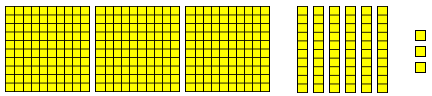
Answer: 38 $^{4}/_{15}$

511 ÷ 35 becomes

1 4 . 6
35 | 5 $1^{16}1$ $^{21}0$

Answer: 14.6

363 ÷ 3 = 121 (3 | 363)



**For interactive representations try mathsframe.co.uk ~ Resources ~ New Mathematics Curriculum (2014) / Addition & Subtraction

Brockwell Junior School CALCULATION POLICY ~ PROGRESSION THROUGH FRACTIONS

Y2 and below

Recognise, find and name a half as one of two equal parts of an object, shape or quantity.

Recognise, find and name a quarter as one of four equal parts of an object, shape or quantity.

Recognise, find, name and write fractions $^1/_3$, $^1/_4$, $^2/_4$ & $^3/_4$ of a length, shape, set of objects or quantity

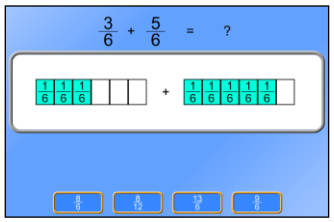
Write simple fractions e.g. $^1/_2$ of 6 = 3 and recognise the equivalence of $^2/_4$ and $^1/_2$

Y3

Add and subtract fractions with the same denominator within one whole
[e.g. $^5/_7 + {}^1/_7 = {}^6/_7$]
supported by concrete representation.

E.g. use fraction towers to count different fraction types.

e.g. interactive 'bar' examples at mathsframe.co.uk (do extend beyond a whole)



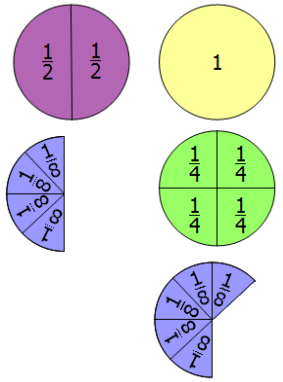
Solve problems that involve adding and subtracting fractions

Y4

Add and subtract fractions with the same denominator including mixed numbers supported by concrete representation.

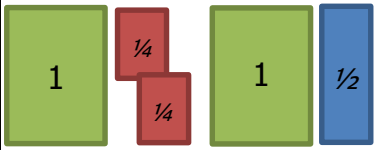
E.g. use fraction action shapes to model counting and adding fractions and wholes.

E.g. $1\,^4/_8 + 2\,^5/_8 = 3\,^9/_8 = 4\,^1/_8$

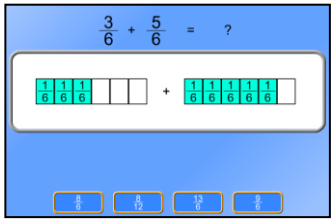


OR

Fraction cards as demonstrated through NCETM video
e.g. $1\,^2/_4 + 1\,^1/_2 = 3$



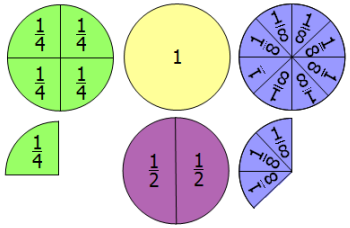
E.g. interactive 'bar' examples at mathsframe.co.uk



Y5

Add and subtract fractions with the same denominator & denominators that are multiples of the same number supported by concrete representation.

E.g. $1\,¼ + 3\,^3/_8$ *such that the ¼ gets swapped for* $^2/_8$.



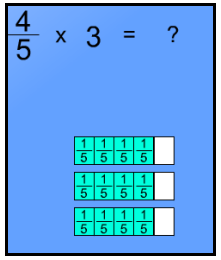
Then developed into recognizing equivalent fractions.

E.g. $1\,^1/_4 + 3\,^3/_8$

$= 1\,^2/_8 + 3\,^3/_8 = 4\,^5/_8$

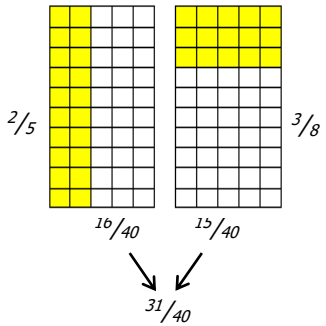
Multiply proper fractions & mixed numbers by whole numbers, supported by materials & diagrams.

E.g. interactive 'repeated bar method' at mathsframe.co.uk

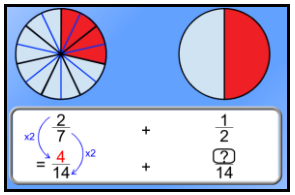


Y6

Add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions and visual representations.
E.g. use of arrays



E.g. use of equivalent fractions interactive examples at mathsframe.co.uk



Multiply simple pairs of proper fractions, writing the answer in its simplest form through visual representation.

Multiply simple pairs of proper fractions, writing the answer in the simplest form. E.g. $^1/_4 \times {}^1/_2 = {}^1/_8$
Start with a rectangle… Split it into quarters…and split each of those into half = $^1/_8$

1/8	1/8	1/8	1/8
1/8	1/8	1/8	1/8

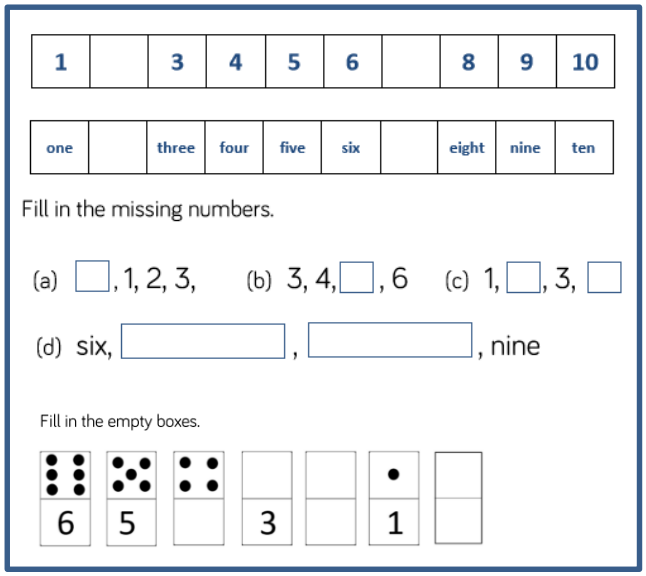
Then as algorithm when patterns have been spotted.
E.g. $½ \times ¼ = {}^1/_8$ *through recognition that denominators have been multiplied.*
Divide proper fractions by whole numbers through visual representation.
e.g. using rectangle method

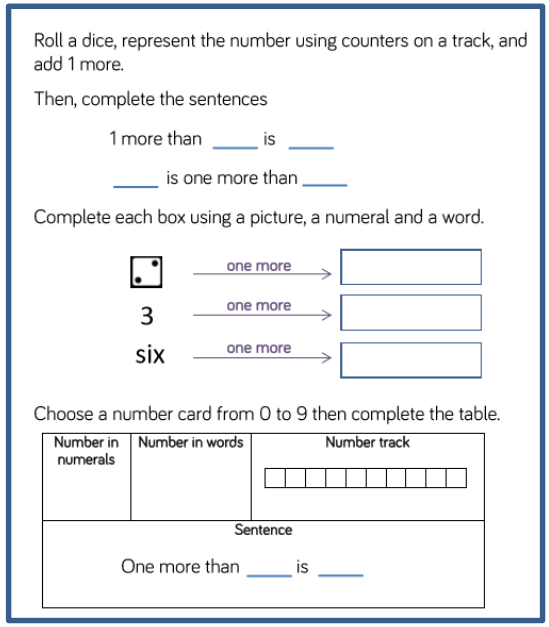
Divide proper fractions, by whole numbers. E.g. $^1/_3 \times 2 = {}^1/_6$
Start with a rectangle… Split it into thirds… and divide those thirds in 2 = $^1/_6$

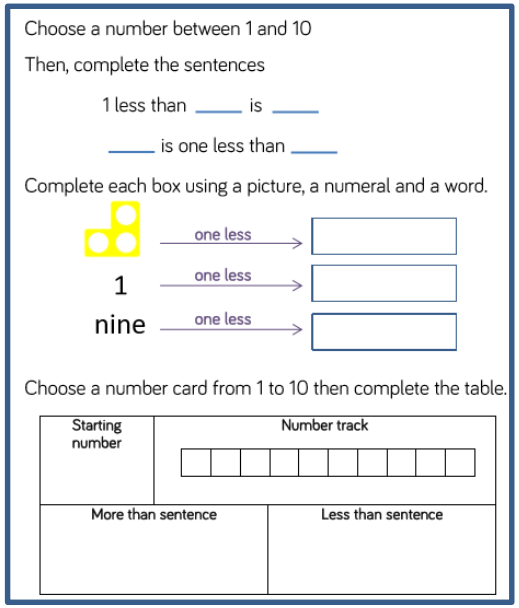
Varied Fluency

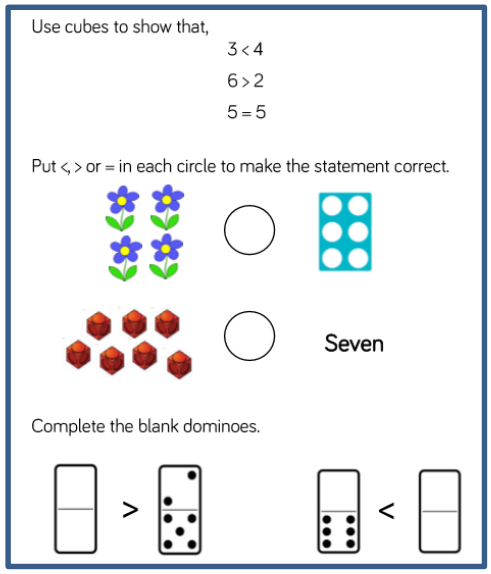
By showing different representations, pupils have a fuller understanding about number. Below are some examples that are used in each year group. These representations are taken from the White Rose Small Steps Guidance and Examples which is available in school.

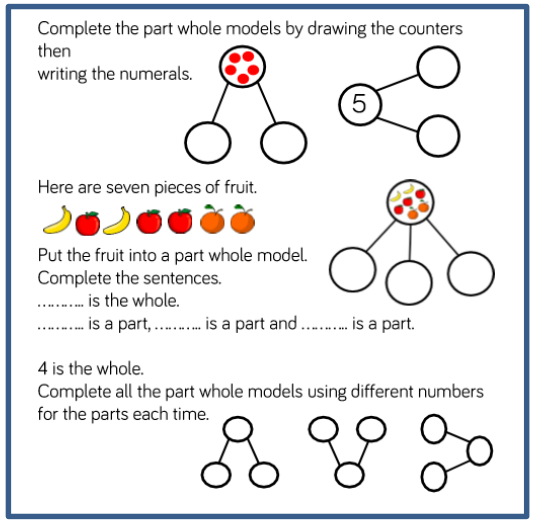
Year 1

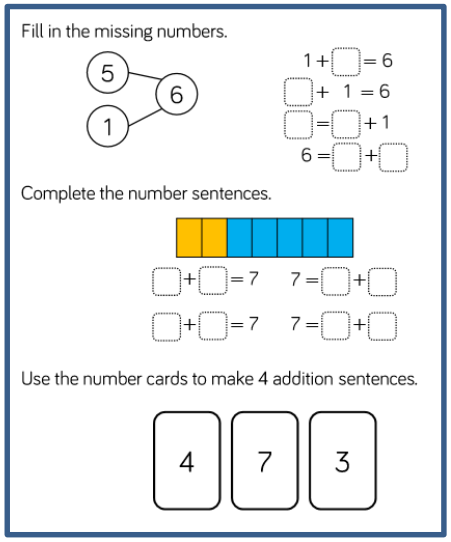












Year 2

Here is part of a bead string.

Complete the sentence.

There are …… tens and …… ones.

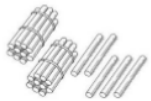
The number is ……

Represent 45 on a bead string.

Match the number to the correct representation.

Three tens and four ones

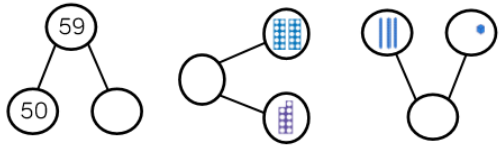
Twenty five



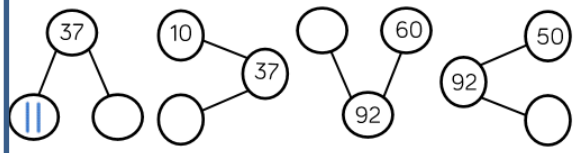
33

Represent 67 in **three different** ways?

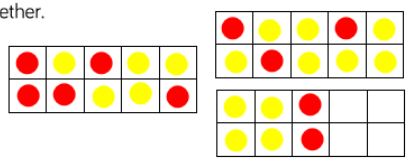
Complete the part whole models.



Complete the part whole models.



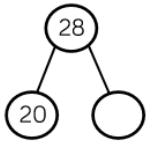
The ten frames represent lemon and strawberry cupcakes.
Draw a part whole model to show how many cupcakes there are altogether.



Match the number sentences to the correct number.

20 + 19	10 + 4	40 + 0	80 + 1
40	14	81	39

Complete the part-whole model and write four number sentences to match.



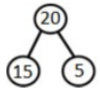
____ + ____ = ____
____ + ____ = ____
____ = ____ + ____
____ = ____ + ____

Hattie has 20 sweets and Noah has 15 sweets.
Represent the total number of sweets:

- With concrete resources
- In a part whole model
- As a number sentence

Using concrete apparatus, can you talk about the relationships between the different flowers?

One relationship shown by this part whole model is
15 + 5 = 20
Can you write all associated fact facts in the sentences below?

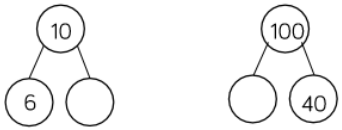


Look at the bar model below. Can you write all of the sentences in the fact family?

I have 3 blue pens and 4 black pens. Together I have 7 pens.
Tom has 30 blue pens and 40 black pens. How many does he have in total?

Use concrete apparatus to show your thinking.

Complete the part whole models below:



Find the missing numbers in the related facts.

5 + 4 = 9
50 + 40 = ☐

8 = 3 + 5
80 = 30 + ☐

4 = 10 − 6
40 = ☐ − 60

Find the sum of 34 and 23

64 + 12 =
4 ones + 2 ones = ☐
6 tens + 1 ten = ☐
☐ tens + ☐ ones = ☐

Hamza has 41 sweets.

Jemima has 55 sweets.

How many sweets do they have altogether?

Year 3

What is the value of the number represented in the place value chart?

Write it in numerals and words.

Complete this place value chart so that it shows 354

What number would this make?

What number is shown in the place value chart?

If one more (10) is added. What number would be shown?

True or false?
The place value grid shows 615

Put <, > or = in the circles to make the statement correct.

Circle the greatest amount in each case.

Nine hundred and two	920
500 and 63	568
7 hundreds and 6 ones	76 tens

Fill in the circle with <, > or =

399 ◯ 501

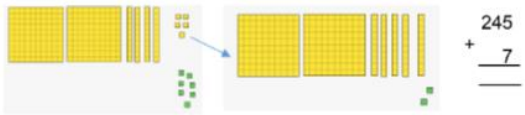
800 ◯ 80 tens

Complete the statements.

600 + 70 + 4 > 600 + ... + 4

Two hundred and five < .....

Solve 245 + 7

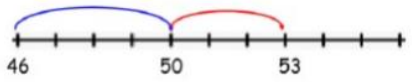


$$\begin{array}{r} 245 \\ +\ \ 7 \\ \hline \end{array}$$

Calculate three hundred and seventy nine add five.

379 + 1 = 380
380 + 4 = 384

Use a number line to calculate 346 + 7



46 + 7 = 53
300 + 53 = 353

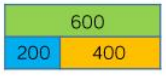
2 ones and 3 ones is equal to 5 ones

2 tens and 3 tens is equal to 5 tens

So 2 hundreds and 3 hundreds is equal to ..... hundreds

Complete each box for 400 + 500

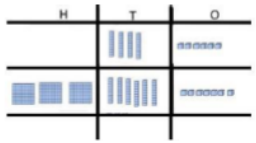
Use the bar model to complete the number sentences.



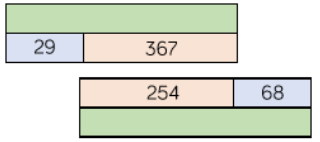
____ + ____ = 600
____ + ____ = 600
____ - ____ = 400
____ - ____ = 200

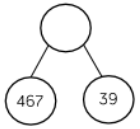
600 = ____ + ____
600 = ____ + ____
400 = ____ - ____
200 = ____ - ____

Represent 46 + 367 using Base 10.



Use column addition to work out:





Use column addition to work out

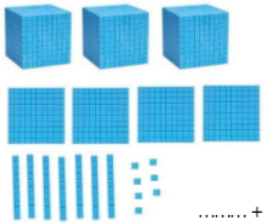
248 + 37

476 + 59

556 − 77

Year 4

Complete the sentences.

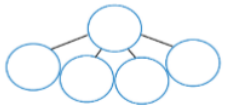


There are …… thousands, …… hundreds, …… tens and …… ones.

The number is ……

…… + …… + …… + …… = ……

Complete the part-whole model for the number represented.



What is the value of the underlined digit in each number?

6,983 9,021

789 6,570

Draw arrows to show where the numbers would be on the number line.

8,750

6,000 9,000

4,100

0 10,000

Estimate the value of each letter.

A B C D

0 10,000

X Y Z

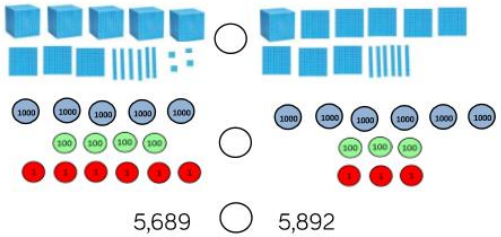
2,000 6,000

Estimate the value of A.

A

6,300 8,490

Fill in the circle using <, > or =



5,689 ◯ 5,892

Circle the smallest amount.

Two thousand, three hundred and ninety seven 3,792

6,000 + 400 + 50 + 6 6,455

9 thousands, 2 hundreds and 6 ones 9,602

Complete the statements.

1,985 > ……

4,203 < 4,000 + …… + 4

Add the place value counters together.

1,000s	100s	10s	1s
1000 1000 1000	100 100	10 10 10 10	1 1
1000 1000	100 100	10	1 1 1

Can you write this as a calculation? (3,242 + 2,213)
Now complete the question 3,242 + 213 in the same way.
What is the same and what's different?
Look at how the place value columns are lined up in the new question.
How is our answer different? Why?

Complete the missing numbers.

4□6□
+2 5□1
□789

Subtract 2,332 from the number below.

Complete this subtraction problem.

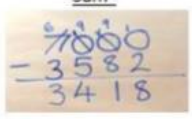
	Thousands	Hundreds	Tens	Ones
	7	6	4	6
-	4	3	3	5

Using a place value grid work out the following.

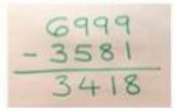
2,348 - 235 = _____
_____ = 4,572 - 2,341
6,582 - 582 = _____
= 7,262 - 7,151

Sam, Lucas and Jemima are solving the calculation
7000 - 3582
Here are their methods.

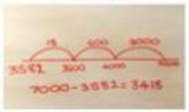
Sam



Lucas



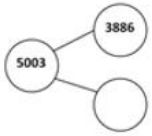
Jemima



Who is correct? Can you explain how each child has reached their answer? Whose method is the most efficient?
Use the different methods to solve 4000 - 2831

Find the missing numbers.
What methods did you use?

3465	
2980	



Year 5

Complete the table.

Start number	Rounded to the nearest 10	Rounded to the nearest 100	Rounded to the nearest 1,000
			
			
DCCLXIX			

For each number, find five numbers that round to it when rounding to the nearest 100

300 | 10,000 | 8,900

Complete the table.

Start number	Nearest 10	Nearest 100	Nearest 1,000
365			
1,242			
	4,770		

A number is shown in the place value chart.

10,000s	1,000s	100s	10s	1s
				

Write the number in figures and in words.

- Ashy adds 10 to this number
- Zack adds 100 to this number
- Isobel adds 1,000 to this number

Write each of their new numbers in figures and in words.

Complete the grid to show the same number in different ways.

Counters	Part whole model
Bar model: 65,048	Number line

Complete the missing numbers.

59,000 = 50,000 + ...

..... = 30,000 + 1,700 + 230

75,480 = ... + 300 + ...

100,000s	10,000s	1,000s	100s	10s	1s

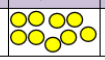
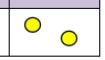
Use counters to make these numbers on the place value chart.

- 32,651
- 456,301
- 50,030

Can you say the numbers out loud?

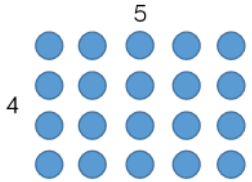
Complete the part whole diagrams.

Katya has the following number.

10,000s	1,000s	100s	10s	1s
				

She adds 4 counters to the hundreds column.
What is her new number?

If you have twenty counters, how many different ways of arranging them can you find? How many factors of twenty have you found? E.g. A pair of factors of 20 are 4 and 5



Circle the factors of 60

9, 6, 8, 4, 12, 5, 60, 15, 45

Which factors of 60 are not shown?

Fill in the missing factors of 24

1 × ☐ ☐ × 12

3 × ☐ ☐ × ☐

What do you notice about the order of the factors?
Use this method to find the factors of 42

Use arrays to find the common factors of 12 and 15
Can we arrange the counters in one row?

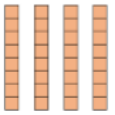
Yes- so they have a common factor of one.
Can we arrange the counters in two equal rows?

2 is a factor of 12 but not of 15 so 2 is not a common factor.
Continue to work through the factors systematically until you find all the common factors.

Fill in the Venn diagram to find the factors of 20 and 24

Where are the common factors of 20 and 24? Can you use a Venn diagram to find the common factors of 9 and 15?

Complete the fact family.



☐ × ☐ = ☐ ☐ ÷ ☐ = ☐

☐ × ☐ = ☐ ☐ ÷ ☐ = ☐

Can you use the same representation to complete the sentences?

____ and ____ are factors of ____

____ is a multiple of ____ and ____

Complete the missing numbers.

3 × ☐ = 21 3 × ☐ = 42 42 ÷ ☐ = 6

☐ ÷ 14 = 6 84 ÷ ☐ = 42 ÷ 6

Use <, > or = to complete the number sentences.

81 ÷ 9 ◯ 24 ÷ 3

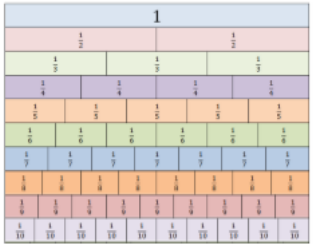
6 × 8 ◯ 16 × 3

Year 6

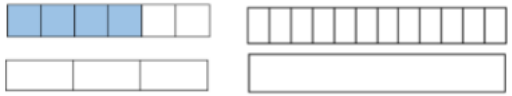
Use the fraction wall to simplify: $\frac{2}{8}$, $\frac{3}{9}$ and $\frac{4}{10}$

Which direction did you move on the fraction wall?

What have the numerator and denominator been divided by?



Use the bar models to simplify the fractions.
Make sure your bar model has fewer equal parts than the original fraction.



$\frac{4}{6} = \frac{\square}{3}$ $\frac{8}{12} = \frac{\square}{\square}$

Use the bar models to show $\frac{1}{4}$ and $\frac{2}{3}$ then complete the sentences.

□ is larger than □

□ is smaller than □

□ < □

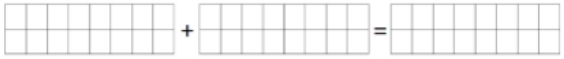
Complete the circles using <, > or =

$\frac{3}{5} \bigcirc \frac{4}{7}$ $\frac{2}{6} \bigcirc \frac{1}{4}$ $2\frac{1}{5} \bigcirc 2\frac{3}{8}$

$\frac{7}{8} \bigcirc \frac{4}{6} \bigcirc \frac{3}{4}$

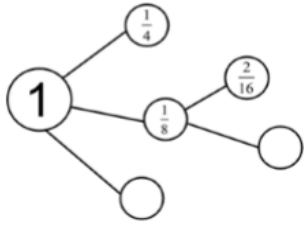
Jen read $\frac{3}{4}$ of her book, Emma read $\frac{3}{10}$ of her book and Amy had read $\frac{4}{5}$ of her book.
Put them in order starting with the person who read the most of their book.

Shade in the diagram to show that $\frac{5}{8} + \frac{3}{16} = \frac{13}{16}$



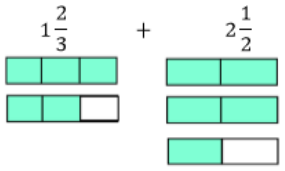
Draw your own diagram to show that $\frac{1}{3} + \frac{2}{9} = \frac{5}{9}$

Complete the part whole model.



Emma uses $\frac{1}{3}$ of her tin of paint on Friday, $\frac{1}{21}$ on Saturday and on Sunday she uses $\frac{2}{7}$.
How much paint does she have left?

Can you split the bar models so each fraction has the same denominator?

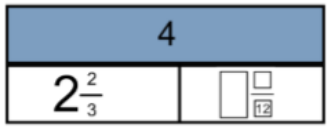


How can you use this information to solve the original calculation?

Complete the calculation.

$\square\frac{\square}{\square} = 3\frac{1}{2} + 1\frac{1}{4}$

Complete the bar model.



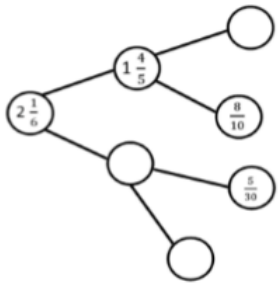
Calculate $3\frac{1}{4} - 1\frac{3}{4}$

$3\frac{1}{4}$ can become $2\frac{5}{4}$

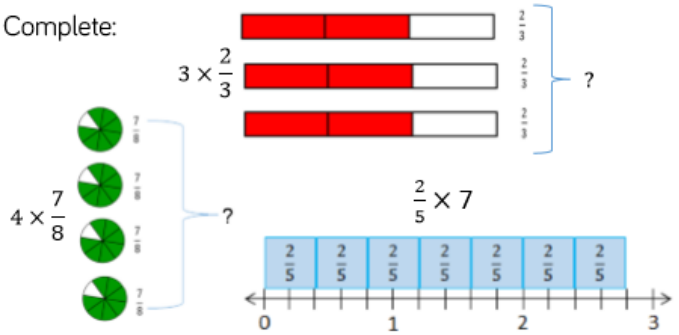
How can you use the equivalent fraction of $2\frac{5}{4}$ to complete the calculation?

Tina has $3\frac{2}{3}$ bags left of bird feed. She uses $1\frac{4}{6}$.
How much will she have left?

Complete the part whole model.



Complete:



Sally and 3 of her friends have $1\frac{2}{3}$ of a chocolate bar each.
How much chocolate do they have altogether?

Complete and then order:

- $6 \times \frac{5}{7}$ $\frac{5}{6} \times 5$ $4 \times \frac{7}{8}$
- $4 \times 2\frac{3}{5}$ $3\frac{4}{9} \times 3$ $5 \times 2\frac{3}{7}$

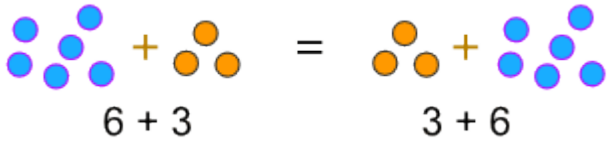
Mathematical Laws

Even though the laws below look confusing, they are the basic rules that govern our teaching at Mathematics. The children won't be taught these laws as shown below but they will be taught the application of these laws through daily maths learning. It is worth noting how the laws work for multiplication and addition but not always for division or subtraction.

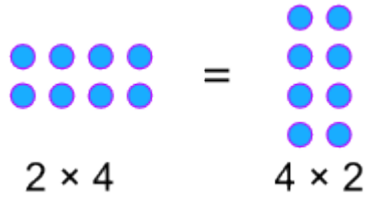
Commutative Laws:	$a + b = b + a$ $a \times b = b \times a$
Associative Laws:	$(a + b) + c = a + (b + c)$ $(a \times b) \times c = a \times (b \times c)$
Distributive Law:	$a \times (b + c) = a \times b + a \times c$

Commutative Laws

The "Commutative Laws" say we can **swap numbers** over and still get the same answer...
...when we **add**: $a + b = b + a$
Example:



... or when we **multiply**: $a \times b = b \times a$
Example:



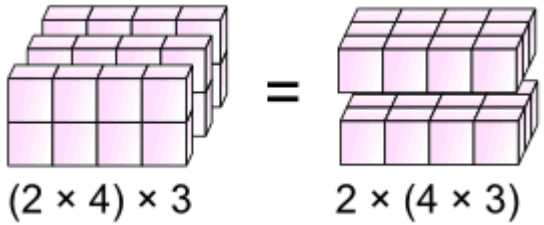
Associative Laws

The "Associative Laws" say that it doesn't matter how we group the numbers (i.e. which we calculate first) ..... when we **add**:

$$(a + b) + c = a + (b + c)$$

... or when we **multiply**:

$$(a + b) + c = a + (b + c)$$



Examples:

This:	**(2 + 4)** + 5 **= 6** + 5 **= 11**
Has the same answer as this:	2 + **(4 + 5)** = 2 + **9 = 11**

This:	**(3 × 4)** × 5 = **12** × 5 = **60**
Has the same answer as this:	3 × **(4 × 5)** = 3 × **20** = **60**

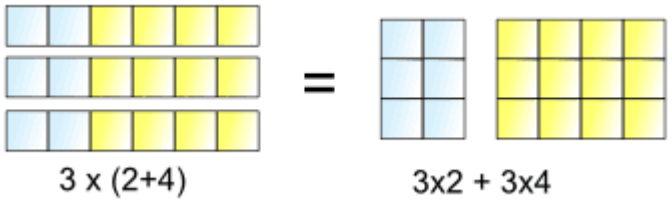
Uses: Sometimes it is easier to add or multiply in a different order:

What is 19 + 36 + 4? 19 + 36 + 4 = 19 + **(36 + 4)** = 19 + **40** = 59

Or to rearrange a little: What is 2 × 16 × 5? 2 × 16 × 5 **= (2 × 5)** × 16 **= 10** × 16 = 160

Distributive Law

The "Distributive Law" needs careful attention. This is what it lets us do:



3 lots of **(2 + 4)** is the same as **3 lots of 2** plus **3 lots of 4**

So, the **3×** can be "distributed" across the **2 + 4**, into **3 × 2** and **3 × 4**

And we write it like this: $a \times (b + c) = a \times b + a \times c$

Uses: Sometimes it is easier to break up a difficult multiplication:

Example: What is 6×204? $6 \times 204 = 6 \times 200 + 6 \times 4 = 1{,}200 + 24 = 1{,}224$

Or to combine: Example: What is $16 \times 6 + 16 \times 4$? $16 \times 6 + 16 \times 4 = 16 \times \mathbf{(6 + 4)} = 16 \times \mathbf{10} = 160$

We can use it in subtraction too: Example: $26 \times 3 - 24 \times 3$ $6 \times 3 - 24 \times 3 = \mathbf{(26 - 24)} \times 3 = 2 \times 3 = 6$

We could use it for a long list of additions, too:

Example: $6 \times 7 + 2 \times 7 + 3 \times 7 + 5 \times 7 + 4 \times 7$ $\mathbf{6} \times 7 + \mathbf{2} \times 7 + \mathbf{3} \times 7 + \mathbf{5} \times 7 + \mathbf{4} \times 7 = \mathbf{(6 + 2 + 3 + 5 + 4)} \times 7 = \mathbf{20} \times 7 = \mathbf{140}$